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INSTANTANEOUS PRODUCTION GROWTH IN A DYNAMIC ECONOMY

CRECIMIENTO INSTANTÁNEO DE LA PRODUCCIÓN EN UNA ECONOMÍA DINÁMICA

CRESCIMENTO INSTANTÂNEO DA PRODUÇÃO EM UMA ECONOMIA DINÂMICA

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Abstract

The purpose of this study is to measure the *average* and *instantaneous* rates of production growth and to provide new formulas for measuring the contribution to and share of resources in that growth. This is the key idea in a *new method of attack* as suggested by Cobb and Douglas—as of yet not solved by economic growth theory—to demonstrate that distribution processes are modeled on those of the production of value. We intend to validate the strength of the proposed model by systematizing and analyzing production, investment and employment data from a dynamic economy, i.e., the United States of America, provided by the Bureau of Economic Analysis (BEA) and the Federal Reserve (Fed) for the years 2012 to 2022, the results of which can be found in the Appendix. We conclude that greater convergence with equity between labor and capital contribution and share in production growth results when the elasticity of its composition (μ) is between ½ and 1. Outside of this range, we see divergence with inequity adverse to labor.

Our model could improve the formulation and execution of macroeconomic policies (fiscal, monetary and trade) and promote greater convergence/equity in the contribution to and share of resources in production growth.

Key Words: Resource composition; Resource contribution and share; Shrift; Capital accumulation.

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Resumen

El propósito de este estudio es medir la *tasa media* e *instantánea* del crecimiento económico y proveer nuevas fórmulas para medir la contribución y participación de los recursos en el mismo. Esta es la idea central en *un nuevo método de ataque* sugerido por Cobb y Douglas (1928), con objeto de demostrar que los procesos de distribución se modelan por los de producción de valor. Se pretende validar la fortaleza del modelo propuesto al sistematizar y analizar datos de producción, inversión y empleo de una economía dinámica como la de los Estados Unidos de América, provistos por la Oficina de Análisis económico (BEA) y la Reserva Federal (Fed) durante 2012-2022, cuyos resultados aparecen en el Apéndice. Se concluye que la mayor convergencia con equidad entre contribución y participación del trabajo y el capital en el crecimiento económico se presenta cuando la elasticidad de su composición (μ) se ubica entre ½ y 1. Fuera de ese rango se presenta divergencia con inequidad adverso al empleo.

El modelo podría mejorar la formulación y ejecución de políticas macroeconómicas (fiscal, monetaria, comercial) y promover mayor convergencia/equidad en la contribución/participación de los recursos en el crecimiento económico.

Palabras Clave: Composición de los recursos; Contribución y participación de los recursos; Ahorro; Acumulación de capital.

Resumo

O objetivo deste estudo é medir as taxas médias e instantâneas de crescimento da produção e fornecer novas fórmulas para medir a contribuição e a participação dos recursos nesse crescimento. Essa é a ideia principal de um novo método de ataque, conforme sugerido por Cobb e Douglas - e ainda não resolvido pela teoria do crescimento econômico -, para demonstrar que os processos de distribuição são modelados de acordo com os da produção de valor. Pretendemos validar a força do modelo proposto sistematizando e analisando dados de produção, investimento e emprego de uma economia dinâmica, ou seja, os Estados Unidos da América, fornecidos pelo Bureau of Economic Analysis (BEA) e pelo Federal Reserve (Fed) para os anos de 2012 a 2022, cujos resultados podem ser encontrados no Apêndice. Concluímos que há maior convergência com a equidade entre a contribuição e a participação do trabalho e do capital no crescimento da produção quando a elasticidade de sua composição (μ) está entre ½ e

Fora desse intervalo, observamos divergência com a desigualdade adversa ao trabalho. Nosso modelo poderia melhorar a formulação e a execução de políticas macroeconômicas (fiscais, monetárias e comerciais) e promover

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maior convergência/equidade na contribuição e na participação dos recursos no crescimento da produção.

Palavras-chave: Composição de recursos; Contribuição e participação de recursos; Desvio; Acumulação de capital.

INTRODUCTION

Cobb and Douglas' (1928) suggestions and challenges prompt us to devise *a new method of attack* for resolving those challenges. It involves finding new key formulas to measure variations in resource contribution to production growth and to simplify *the time factor of the process*. An attempt at this was made by Harrod (1939; 1960), Swan (1956), Solow (1956; 1957), Arrow *et al.* (1961) and, more recently, Piketty (2014), among other remarkable authors. The study of resource contribution to and share in production growth aims to demonstrate that the distribution process is modeled on that of the production of value (Villalobos, 2019; 2019a; 2020; 2020a) and (Villalobos, 2022). We base our model on mathematical tools given that *economics* [is] *the most "exact" of the social sciences* (Allen, 1938).

Cobb and Douglas' (1928) suggestions are based on five questions:

a) Can we estimate, within limits, whether [the] increase in production was purely fortuitous, whether it was primarily caused by technique, and the degree, if any, to which it responded to changes in the quantity of labor or capital?
b) May it be possible to determine, […], the *relative* influence upon production of labor as compared with capital?
c) As the proportions of labor to capital changed from year to year, may it be possible to deduce the relative amount added to the total physical product by each unit of labor and capital and what is more important still by the *final* units of labor and capital in [the] respective years?
d) Can we measure the probable slopes of the curves of incremental product which are imputed to labor and to capital and thus give greater definiteness to what is at present purely an hypothesis with no quantitative values attached?
e) [… M]ay we [shed] light upon the question as to whether or not the processes of distribution are modeled at all closely upon those of the production of value?

The objective of this research is to provide a *method* and *techniques of attack* for the problems of resource contribution to and share in production growth. *To begin with,* we need to *regard the fundamental concept in dynamic economics as the rate of increase* […] *that obtains at a given point of time, given the fundamental determinants*, as a result of *the relations between the rates of increase (or decrease) of certain magnitudes in a growing economy* (Harrod, 1960).

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To begin, section one of this paper presents the fundamental mathematical tools to reveal conceptual relations inherent to production, capital and labor. Section two introduces capital and labor *contribution* to production growth, from which we get the relative rate of production growth, and the formulas for measuring the elasticity of resource composition and the elasticity of contribution. After deriving the *potential* rate of production growth, section three deals with the problem of determining the *slope* of the production function, followed by section four, which discusses the share of resources in production growth. Section five then analyzes the thrift and capital accumulation processes and, lastly, section six provides an approach to national accounting. In the section entitled Conclusions, we also provide our conclusions, implications and possible lines of research followed by the references used.

Since the importance of our method of attack depends on its use in interpreting scientific phenomena (Allen, 1938), we considered its applicability to the United States of America's economy (2012-2022), from which some of the relevant results are shown in tables and figures in the Appendix. Our model provides good answers to Cobb and Douglas' five questions, suggestions and challenges, and Harrods' (1939; 1960) hypothesis on dynamic economy. Evaluating the model in a large number of economic sectors or countries might offer additional evidence of its accuracy and strength.

Average **and** ***instantaneous*** **production growth**

Suppose an economy generating different levels of production $[Y_\tau, Y_t]$ at different *instants* $(\tau, t); (\tau < t)$. Each level of production is a function of a precise point in time within a period $[\tau, t]$. The levels of production at those points are:

$$(1)\; Y_\tau = F(\tau)$$

$$(2)\; Y_t = F(t)$$

$F(t)$ can be evaluated at a given point $(\tau, F(\tau))$ by the point $(t, F(t))$ and thus compute *average* changes in production levels in a specific period of time. Mathematically, this refers to the *secant line* through $[\tau, t]$, with *slope* m:

$$(3)\; m_{sec} = \frac{F(t) - F(\tau)}{t - \tau}$$

m_{sec} indicates the *rate of change* of $F(t)$ or the *average velocity* of (Y) over the time period $[\tau, t]$, and as it approaches a *line* called *tangent* to $F(t)$ at τ, it measures the *rate of change* of $F(t)$ at τ or its *derivative* at τ. As t approaches τ the *instantaneous velocity* of (Y) is reached at *each given instant*. Defining $(t = \tau + h)$ where $(h > 0)$ is the allotted time change, close to τ, per equation (2):

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(4) $Y_t = F(\tau + h)$

which inserted into equation (3) results in $m_{sec} = \frac{F(\tau+h)-F(\tau)}{(\tau+h)-\tau}$ and:

(5) $$m_{sec} = \frac{F(\tau + h) - F(\tau)}{h}$$

As the value of $(h \to 0)$, for economic reasons, the *average velocity* of (Y) becomes the slope of the function of production growth; m_{sec} approaches the *instantaneous velocity* and the rate of change of $F(t)$ at (τ) is the *slope* of the *tangent line* m_{tan} at that instant:

(6) $$m_{tan} = \lim_{h \to 0} \frac{F(\tau + h) - F(\tau)}{h}$$

Alternatively, in terms of the derivative:[2]

(7) $$\frac{dY}{dt} = \lim_{h \to 0} \frac{F(\tau + h) - F(\tau)}{h}$$

From $(t = \tau + h)$ it is clear that $[dt = h = (t - \tau)]$ is the range of the time period $[\tau, t]$ provided that $(\tau < t)$. We are interested in measuring *gross* changes on (Y) *starting* from τ; e.g., *quinquennial*, *quarterly*, *year-over-year*. Defining $\acute{Y} = \frac{dY}{dt} = \frac{dY}{h}$ per equation (7) will result in $\acute{Y} = \lim_{h \to 0} \frac{F(\tau+h)-F(\tau)}{h}$ and dividing both sides of this identity by $F(t)$ we get $\frac{\acute{Y}}{F(\tau)} = \lim_{h \to 0} \left[\frac{1}{h} \frac{F(\tau+h)-F(\tau)}{F(\tau)}\right] \therefore \frac{\acute{Y}}{F(\tau)} = \lim_{h \to 0} \left[\frac{1}{h} \frac{F(\tau+h)}{F(\tau)} - 1\right]$ and let $\lambda = \left(\frac{F(\tau+h)}{F(\tau)} - 1\right)$ be the *relative rate of production growth* at $(\tau + h)$-instant, so that $\frac{\acute{Y}}{F(\tau)} = \lim_{h \to 0} \frac{1}{h} \lambda$ from which:

(8) $$\acute{Y} = \lim_{h \to 0} \frac{1}{h} \lambda F(\tau)$$

2 "…the systematic evaluation of derivatives is not an end in itself. We evaluate derivatives only because they are useful to us in the application of mathematical methods in the natural or social sciences […] We have, therefore, to learn to evaluate derivatives easily before we can apply them fruitfully in economics." (Allen, 1938, p. 160).

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To state it less formally: from functions (1) and (4) we obtain $\acute{Y} = \frac{1}{h}(Y_t - Y_\tau) \therefore h\acute{Y} = (Y_t - Y_\tau)$ and after dividing both sides by Y_τ, we get $\frac{\acute{Y}}{Y_\tau}h = \left(\frac{Y_t}{Y_\tau} - 1\right)$ and if $\lambda = \left(\frac{Y_t}{Y_\tau} - 1\right)$:

$$\acute{Y} = \frac{1}{h}\lambda Y_\tau \quad (9)$$

By way of example, if we evaluate this equation for a *year-over-year* period $(h = 1)$ we get:

$$\acute{Y} = \lambda Y_\tau \quad (10)$$

At $(h = 1)$, $Y_t = Y_\tau + \acute{Y} = (1 + \lambda)Y_\tau$; if $\left\{0 \leq \frac{Y_t}{Y_\tau} < 1\right\}$ then $(-\lambda)$.[3]

Following the same method with respect to the amount (*real value*) of capital and labor effectively related to production growth at each instant, we get:

$$K_t = F(t);\ \acute{K} = kK_\tau \quad (11)$$

$$L_t = F(t);\ \acute{L} = nL_\tau \quad (12)$$

Where $(\acute{K}, \acute{L})$ measure the average change on (K, L), respectively, at their relative rates of change (k, n) from instant to instant, which could be negative. Production growth could occur at changes in the scale at which resources, influenced by technological changes, are technically combined at each instant. This means that (λ, k, n) could differ at each instant due to economic circumstances, government policies, innovations or radical technological changes shifting resource composition in production processes.

Resource *contribution* to production growth

To study this phenomenon, we must reveal, *ceteris paribus,* how (Y) is determined by (K, L), following the *new method of attack* we explained above. Paraphrasing Allen (1938), *it is important to devise a method of measuring the rate at which* Y_τ *changes when* K_t *changes*. Let $Y_t = F(K_t)$ be the level of production attributed to capital alone at a specific t-instant so that

3 Per function (7), after replacing $\acute{Y} = \frac{dY}{h}$ so that $dY = \lim_{h\to 0} F(\tau + h) - F(\tau)$ and at $h = 0, dY = 0$ due to $\lambda Y_\tau = F(\tau + 0) - F(\tau)$, then $\lambda = (Y_\tau - Y_\tau)/Y_\tau$ and $\lambda = 0$, meaning that relative changes in the level of production might occur at time changes.

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(13) $$\frac{dY}{dK} = \lim_{\acute{K}\to 0} \frac{F\left(K_\tau + \acute{K}\right) - F(K_\tau)}{\acute{K}}$$

Note that the gross change in the level of capital tends to zero $\left(\acute{K} \to 0\right)$ and so per equation (11):

(14) $$\frac{dY}{dK} = \lim_{k\to 0} \frac{F[(1+k)K_\tau] - F(K_\tau)}{kK_\tau}$$

This result affirms that if the *instantaneous relative* rate of change of capital tends to zero $(k \to 0)$ at $(h = t - \tau)$ so that at $(h = 0; t = \tau)$, and it is presumed that $k \to 0; \lambda \to 0$; we are measuring a specific τ-point at which $\lambda = 0$ when $k \geq 0$ generates Y_τ, meaning $Y_\tau = F(K_\tau)$.[4]

Per previous definitions,

(15) $$\frac{\lambda Y_\tau}{kK_\tau} = \lim_{k\to 0} \frac{F[(1+k)K_\tau] - F(K_\tau)}{kK_\tau}$$

andsimplifying, $\lambda Y_\tau = \lim_{k\to 0} [kF(K_\tau)]$ revealingthatat $(h \to 0); (k \to 0);\ (\lambda \to 0)$, and thus:

(16) $$\lambda_K = \lim_{k\to 0} \left[kF\left(\frac{K_\tau}{Y_\tau}\right)\right]$$

Assuming $(h > 0)$, we can expect (k) to be a negative or positive value or even zero, because $\frac{dK}{dt} = \lim_{h\to 0} \frac{F(\tau+h)-F(\tau)}{h} \therefore \acute{K} = \lim_{h\to 0} \frac{1}{h} kK_\tau$ to have $h\acute{K} = kK_\tau$ and provided that $(\mathrm{h} > 0; t > \tau)$ such as $\left(h = 1; \acute{K} = kK_\tau\right)$ and for different circumstances, negative, positive or zero k-values could occur. Let $(1-\alpha) = F\left(\frac{K_\tau}{Y_\tau}\right)$ be the *average* contribution of capital to production growth, and so to find the *relative* rate at which capital contributes to production growth at a given τ:

(17) $$\lambda_K = (1-\alpha)k$$

4 That $[(k,n) \lesseqgtr 0; (h > 0; t > \tau)]$ does not necessarily yield $\lambda = 0$ considering production processes as *continuous*, due to technological changes and technical and scientific knowledge at each given instant, augmenting capital and labor productivity, among other circumstances—natural, political, cyclical, etc.—, so: $\lambda_{h>0} \lesseqgtr \lambda_\tau$.

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As $k \to 0,\ \lambda_K$, *seems to approach the limiting value* $(1-\alpha)$ and $(1-\alpha)=\frac{\lambda_K}{k}$ is *the elasticity of the product with respect to small changes in capital alone*, equal to its *average* contribution at the instant $Y_\tau = F(K_\tau)$. Differentiating Cobb and Douglas' function $Y_\tau = F(L_\tau^\alpha K_\tau^{1-\alpha})$ in terms of (K) gives us $\frac{\partial Y}{\partial K} = f[L_\tau^\alpha\ (1-\alpha)K_\tau^{-\alpha}]$ and $\frac{\lambda Y_\tau}{kK_\tau} = f[L_\tau^\alpha\ (1-\alpha)K_\tau^{-\alpha}] \therefore \lambda Y_\tau = f[L_\tau^\alpha\ (1-\alpha)K_\tau^{-\alpha} kK_\tau] \therefore \lambda Y_\tau = f[(1-\alpha)kL_\tau^\alpha\ K_\tau^{1-\alpha}]$ from which we get equation (17).

Let $Y_\tau = F(L_\tau)$ be the level of production based on labor alone, such that:

$$\text{(18)}\quad \frac{dY}{dL} = \lim_{\acute{L}\to 0} \frac{F\left(L_\tau + \acute{L}\right) - F(L_\tau)}{\acute{L}}$$

per equation (12):

$$\text{(19)}\quad \frac{dY}{dL} = \lim_{n\to 0} \frac{F[(1+n)L_\tau] - F(L_\tau)}{nL_\tau}$$

and per the above definitions,

$$\text{(20)}\quad \frac{\lambda Y_\tau}{nL_\tau} = \lim_{n\to 0} \frac{F[(1+n)L_\tau] - F(L_\tau)}{nL_\tau}$$

As we did for capital, the *instantaneous relative* rate of change of labor *tends* to zero $(n \to 0)$ at $(h = t - \tau)$ and if $(h = 0; t = \tau)$ suggesting that $n \to 0; \lambda \to 0;\ Y_\tau = F(L_\tau)$. After simplifying equation (20), $\lambda Y_\tau = \lim_{n\to 0}[nF(L_\tau)]$:

$$\text{(21)}\quad \lambda_L = \lim_{n\to 0}\left[nF\left(\frac{L_\tau}{Y_\tau}\right)\right]$$

Recall that $(1-\alpha) = F\left(\frac{K_\tau}{Y_\tau}\right)$ is the *average* contribution of capital alone to production growth, which is a fraction of total resource contribution. So, $\left[1 = F\left(\frac{K_\tau}{Y_\tau}\right) + F\left(\frac{L_\tau}{Y_\tau}\right)\right]$ and $1 = (1-\alpha) + F\left(\frac{L_\tau}{Y_\tau}\right)$ then $\alpha = F\left(\frac{L_\tau}{Y_\tau}\right)$ is the *average* contribution of labor alone to production growth. Inserting this result in function (21) will result in the relative rate of labor contribution λ_L to production growth:

$$\text{(22)}\quad \lambda_L = \alpha n$$

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And $\alpha = \frac{\lambda_L}{n}$ is *the elasticity of the product with respect to small changes in labor alone*, equal to its *average* contribution. Per the derivative of Cobb and Douglas' function $Y_\tau = F(L_\tau^\alpha K_\tau^{1-\alpha})$ in terms of (L); $\frac{\partial Y}{\partial L} = f(\alpha L_\tau^{\alpha-1} K_\tau^{1-\alpha})$ so $\frac{\lambda Y_\tau}{n L_\tau} = f(\alpha L_\tau^{\alpha-1} K_\tau^{1-\alpha}) \therefore \lambda Y_\tau = f(\alpha n L_\tau^\alpha K_\tau^{1-\alpha})$, which in turn results in function (22).

If we combine functions (17) and (22), we get the relative rate of *potential* production growth:

$$(23)\ \lambda = \lambda_L + \lambda_K = \alpha n + (1-\alpha)k$$

The relative *rate of increase* λ *that we obtain at a given point in time, given the fundamental determinants*, results from *the relations between the rates of increase (or decrease) of certain magnitudes* (k, n) *in a growing economy*, as Harrod (1960) anticipated. Paraphrasing Allen (1938), *the* relative *rate of increase* λ *is a perfect measure for any actual* relative *rates of increase* (k, n) in (K, L), *however small*. Additionally, we simplified *the time factor of the process* in the dynamic economy as Cobb and Douglas (1928) expected. To evaluate the *instantaneous relative* rate of production growth, let's assume $\lambda = 0$, given $(h = t - \tau)$ and thus $(h = 0; t = \tau)$, such that $-\alpha n = (1-\alpha)k$ and $\left| -\frac{n}{k} \right|_{h=0} = \frac{1-\alpha}{\alpha}$ and defining $\mu = \left| -\frac{n}{k} \right|_{h=0} \geq 0$:

$$(24)\ \mu = \frac{1-\alpha}{\alpha}$$

μ is the *elasticity of resource composition* at each *instantaneous* τ at which we can compute the *instantaneous* increment in production as the contribution of labor and capital to production growth converges.

μ is decisive in measuring *instantaneous* production growth according to changes in capital and labor levels at τ-instant, because it allows us to assess the elasticity of the product with respect to relative changes in labor α and capital $(1-\alpha)$ at $(h = 0; t = \tau)$ to calculate $Y_\tau = F(K_\tau, L_\tau)$ or $Y_\tau = F(L_\tau^\alpha K_\tau^{1-\alpha})$ or any other production function:

$$(25)\ \alpha = \frac{1}{1+\mu}$$

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α is not constant; it changes as μ fluctuates as it approaches the τ-instant, as anticipated by Cobb and Douglas (1928) and α introduces the *perfect* evaluation of λ at the *instantaneous* $\{\mu: (0, \infty)\}$. Let $\alpha = F(\mu)$ such that:

$$\lim_{\mu \to 0} F(\mu) = \lim_{\mu \to 0} \left(\frac{1}{1+\mu} \right) = 1 \quad (26)$$

$$\lim_{\mu \to \infty} F(\mu) = \lim_{\mu \to \infty} \left(\frac{1}{1+\mu} \right) = 0 \quad (27)$$

We claim that $\{\alpha: (0{,}1)\}$ and at this range α descends asymptotically as $\mu > 0$: $\lim_{\mu \to 1} \left(\frac{1}{1+\mu} \right) = \frac{1}{2}; \lim_{\mu \to 2} \left(\frac{1}{1+\mu} \right) = \frac{1}{3} \ldots$

Slope of the production growth function

Per equation (24) $\mu = \frac{n}{k} = \frac{1-\alpha}{\alpha}$ and the above definitions $k = \frac{\acute{K}}{K_\tau}, n = \frac{\acute{L}}{L_\tau}$ such that $\mu = \frac{\acute{L}/L_\tau}{\acute{K}/K_\tau}$ then $\frac{\acute{L}/L_\tau}{\acute{K}/K_\tau} = \frac{1-\alpha}{\alpha} \therefore \frac{\acute{K}}{\acute{L}} = \frac{\alpha}{1-\alpha} \frac{K_\tau}{L_\tau}$, and if $\kappa_\tau = \frac{K_\tau}{L_\tau}$ is the resource composition at each given instant, it follows that $\frac{\acute{K}}{\acute{L}} = \frac{\alpha}{1-\alpha} \kappa_\tau$. We know that $\frac{dK}{dt} = \acute{K} = \lim_{h \to 0} \frac{F(\tau+h)-F(\tau)}{h}$ and $\frac{dL}{dt} = \acute{L} = \lim_{h \to 0} \frac{F(\tau+h)-F(\tau)}{h}$ provide the *instantaneous* change of those resources at $h \to 0$. Let $m = \frac{\acute{K}}{\acute{L}}$ such that:

$$m = \frac{1}{\mu} \kappa_\tau \quad (28)$$

and m is the *instantaneous slope* of the production function. At $(\mu = 1;\ k = n)$ we get $m = \kappa_\tau$ meaning that the *instantaneous slope* of the production function converges with the *instantaneous slope* of resource composition which we term *constant return to scale*; if all resources increase at the same proportion, so does production growth and labor contribution will be $\alpha = \frac{1}{2}$.

If $[\mu > 1;\ (n > k)]$, then higher production functions become flatter as their slope diminishes, the slope of κ_τ tends downward and $\alpha \to 0$. At $[\mu < 1;\ (n < k)]$ higher production functions will become vertical and the slope of κ_τ will rise and $\alpha \to 1$. Hence μ oscillations around κ_τ will trace a path of convergence/divergence mapped by (m) points at each given instant. Therefore, the slope of the relative rate of incremental product is not *purely a hypothesis with no quantitative values attached* (Cobb & Douglas, 1928).

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Resource *share* in production growth

Equations (17) and (22) express the relative labor and capital *contribution* to production growth. We are also interested in measuring its *share* in production growth—the fraction of λ going to workers and to capitalists. If production growth is proportional to the increase in resources, then labor faces the condition $\alpha\lambda = \alpha n$ and capital $(1-\alpha)\lambda = (1-\alpha)k$, and function (23) will show convergence with equity between labor and capital; $(\lambda = n = k)$ and $\left(\mu = 1, \alpha = \frac{1}{2}\right)$ such that $m = \kappa_\tau$. This can be derived from Cobb and Douglas' production function $Y_t = F(L_t^\alpha K_t^{1-\alpha})$: $\frac{\partial Y}{\partial L} = f(\alpha L_\tau^{\alpha-1} K_\tau^{1-\alpha}) \therefore \frac{\partial Y}{\partial L} = f\left[\alpha\left(\frac{K_\tau}{L_\tau}\right)^{1-\alpha}\right] \therefore$

$$\frac{\partial Y}{\partial L} = f(\alpha\kappa_\tau{}^{1-\alpha}) \tag{29}$$

is the relative contribution of labor to production growth. Then, $\frac{\partial Y}{\partial K} = f[L_\tau^\alpha(1-\alpha)K_\tau^{-\alpha}] \therefore \frac{\partial Y}{\partial K} = f\left[(1-\alpha)\left(\frac{L_\tau}{K_\tau}\right)^\alpha\right] \therefore$

$$\frac{\partial Y}{\partial K} = f\left[(1-\alpha)\left(\frac{1}{\kappa_\tau}\right)^\alpha\right] \tag{30}$$

is the relative contribution of capital to production growth.[5] Equating these two functions we get:

$$\kappa_\tau = \frac{1-\alpha}{\alpha} \tag{31}$$

the counterpart of equation (24).

These outcomes can also be obtained by inserting $K_\tau = \kappa_\tau L_\tau$ into function (16) and so $\lambda_K = kF\left(\kappa_\tau \frac{L_\tau}{Y_\tau}\right) \therefore \lambda_K = \alpha\kappa_\tau k \therefore \frac{\lambda_K}{k} = \alpha\kappa_\tau$ and given that $(1-\alpha) = \frac{\lambda_K}{k}$ then $(1-\alpha) = \alpha\kappa_\tau \therefore \kappa_\tau = \frac{1-\alpha}{\alpha}$. In addition, replacing $L_\tau = \frac{1}{\kappa_\tau}K_\tau$ in function (21) will result in $\lambda_L = nF\left(\frac{1}{\kappa_\tau}\frac{K_\tau}{Y_\tau}\right) \therefore \lambda_L = (1-\alpha)n\frac{1}{\kappa_\tau} \therefore \frac{\lambda_L}{n} = (1-\alpha)\frac{1}{\kappa_\tau}$ and if $\alpha = \frac{\lambda_L}{n}$ then $\kappa_\tau = \frac{1-\alpha}{\alpha}$ as in equation (31). These procedures reveal that at $(\lambda = n = k)$ convergence with equity in production growth might occur, for which:

5 See Cobb and Douglas (1928, pp. 156-157).

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$$\alpha = \frac{1}{1+\kappa_\tau} \quad (32)$$

These results suggest a counterpart for equation (25) and tell us that $\kappa_\tau = \mu$ might denote convergence with equity—the possibility of (λ, n, k) growing proportionally. It means that (κ_τ, μ) are conjugated variables and, therefore $\kappa_\tau \neq \mu$, is feasible depending on *centripetal* and *centrifugal* forces, specifically business cycles, innovations, technological changes and government policies. We are tempted to claim that μ might express the propensity of *exploitation* induced by some global forces on economic growth; *within limits*, changes *in production are not purely fortuitous*.

Let us substitute α with β in Cobb and Douglas' production function, transmuting it as follows:

$$Y_\tau = F\left(L_\tau^{\beta} K_\tau^{1-\beta}\right) \quad (33)$$

β is the elasticity of the *share* of labor in production growth, which could differ from α. Given $\kappa_\tau = \frac{K_\tau}{L_\tau}$ two possibilities can be depicted: $K_\tau = \kappa_\tau L_\tau$ and $L_\tau = \frac{1}{\kappa_\tau} K_\tau$. Replacing $K_\tau = \kappa_\tau L_\tau$ in function (33) results in:

$$Y_\tau = F(\kappa_\tau^{1-\beta} L_\tau) \quad (34)$$

And after differentiating:

$$\lambda_L = \beta n \kappa_\tau^{1-\beta} \quad (35)$$

where λ_L represents the potential relative share of labor (relative wage) in production growth. Inserting $L_\tau = \left(\frac{1}{\kappa_\tau} K_\tau\right)$ into function (33) will result in: $Y_t = F\left[\left(\frac{1}{\kappa_\tau}\right)^{\beta} K\right]$ and after differentiating:

$$\lambda_K = (1-\beta) k \left(\frac{1}{\kappa_\tau}\right)^{\beta} \quad (36)$$

where λ_K is the potential relative share of capital (relative income) in production growth. By equating equation (35) to (36):

$$\mu = \frac{1-\beta}{\kappa_\tau \beta} \quad (37)$$

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which appears to be similar to equation (24) and thus compared to equation (31), nevertheless:

$$\beta = \frac{1}{1+\mu\kappa_\tau} \quad (38)$$

providing that *labor contribution* to production growth α might differ from *labor share* β in production growth. In this case, the condition will show divergence with inequity because of oscillations of μ around κ_τ.

The same result can be derived from Solow's (1956) straightforward production function:

$$Y_\tau = F(K_\tau, L_\tau) \quad (39)$$

which differentiated gives us $\lambda Y_\tau = f(kK_\tau, nL_\tau)$ and after inserting $K_\tau = \kappa_\tau L_\tau$ we get $\lambda Y_\tau = f(k\kappa_\tau, n)L_\tau$ and thus:

$$\lambda_L = \alpha f(k\kappa_\tau, n) \quad (40)$$

By inserting $L_\tau = \frac{1}{\kappa_\tau}K_\tau$ we obtain $\lambda Y_\tau = f\left(k, n\frac{1}{\kappa_\tau}\right)K_\tau$ from which:

$$\lambda_K = (1-\alpha)f\left(k, n\frac{1}{\kappa_\tau}\right) \quad (41)$$

Equalizing these two functions will yield equation (31) and through it (32). Moreover, introducing $K_\tau = \kappa_\tau L_\tau$ into function (39) we get:

$$Y_\tau = F(\kappa_\tau, 1)L_\tau \quad (42)$$

And after differentiating and per the definition $\beta = \frac{L_\tau}{Y_\tau}$:

$$\lambda_L = \beta f(\kappa_\tau, 1)n \quad (43)$$

Then, we insert $L_\tau = \frac{1}{\kappa_\tau}K_\tau$ into function (39) resulting in:

$$Y_\tau = F\left(1, \frac{1}{\kappa_\tau}\right)K_\tau \quad (44)$$

And by differentiating and defining $(1-\beta) = \frac{K_\tau}{Y_\tau}$:

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$$\lambda_K = (1 - \beta) f\left(1, \frac{1}{\kappa_\tau}\right) k \quad (45)$$

By equating functions (43) and (45), we get equation (37) and through it (38).

Let $\beta = F(\mu\kappa_\tau)$ and stating that:

$$\lim_{\mu\kappa_\tau \to 0} F(\mu) = \lim_{\mu \to 0}\left(\frac{1}{1 + \mu\kappa_\tau}\right) = 1 \quad (46)$$

$$\lim_{\mu\kappa_\tau \to \infty} F(\mu) = \lim_{\mu \to \infty}\left(\frac{1}{1 + \mu\kappa_\tau}\right) = 0 \quad (47)$$

And if $\{\beta: (0,1)\}, \beta$, β declines asymptotically as : $\mu\kappa_\tau > 0$: $\lim_{\mu\kappa_\tau \to 1}\left(\frac{1}{1+\mu\kappa_\tau}\right) = \frac{1}{2}$; $\lim_{\mu\kappa_\tau \to 2}\left(\frac{1}{1+\mu\kappa_\tau}\right) = \frac{1}{3}$ ... . This analysis exposes the possibility of $(\lambda_K \neq \lambda_K; \lambda_L \neq \lambda_L)$, which characterizes *divergence/inequity* in production growth distribution (Figure 3). Based on the above developments, effective resource contribution and share in production growth can be computed by substituting (n, k) for (λ) in equations (17) and (22) and (35) or (43) and (36) or (45), and multiplying the result by (Y_τ) to get (γ_L, γ_K) we name the effective resource contribution and $(\acute{y}_L, \acute{y}_K)$ the effective share. Total labor and capital contribution are $(\dot{Y}_L, \dot{Y}_K)$ while their respective shares are $(\ddot{Y}_L, \ddot{Y}_K)$.

Thrift and capital accumulation

Let:

$$K_\tau = F(sY_\tau) \quad (48)$$

be the equation expressing that capital is accumulated at each given instant as a fraction (s) of Y_τ. Therefore, the stock of capital added to each instant can be obtained as follows:

$$\frac{dK_\tau}{dY_\tau} = \lim_{\acute{Y} \to 0} \frac{F(sY_\tau + s\acute{Y}) - F(sY_\tau)}{\acute{Y}} \quad (49)$$

After simplifying, we get $kK_\tau = s\lambda Y_\tau \therefore (1 - \alpha)k = s\lambda$, where λ is the *potential* production growth rate, while k is the *effective* relative increment in capital, so we must replace λ with the *effective* production growth rate (γ) given by computed data from countries or companies, to get:

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$$s = (1-\alpha)\frac{k}{\gamma} \quad (50)$$

By replacing this equivalence in function (23) we find that:

$$\lambda = \alpha n + s\gamma \quad (51)$$

which is similar to Swan's (1956) equation $y = sr + \beta n$ in footnote 5. Assuming $\lambda = 0$ and due to $\mu = \frac{n}{k} \therefore n = \mu k$ we get $\alpha\mu k = s\gamma$ and:

$$s = \alpha\mu\frac{k}{\gamma} \quad (52)$$

or simply $s = \alpha\frac{n}{\gamma}$. This condition establishes the amount of capital needed at τ-instant to continue the production process at a *required* level, which also implies the $s\gamma$ required to satisfy (n, k) for Keynes' (1936) *aggregated demand* and Harrod's (1939) *natural* rates of growth (Villalobos, 2020a). So (s) is determined by capital and labor contribution to production growth and μ at τ.

The same result is reached with equation (23) if we replace $n = \mu k$ such that $k = s\frac{\gamma}{1-\alpha}$ and thus $\lambda = \alpha\mu k + (1-\alpha)k \therefore \lambda = [\alpha\mu + (1-\alpha)]k$. After this, $\lambda = \frac{\alpha\mu+(1-\alpha)}{(1-\alpha)}s\gamma$:

$$s = \frac{1-\alpha}{\alpha\mu + (1-\alpha)}\frac{\lambda}{\gamma} \quad (53)$$

This last equation is more accurate because of the relationship between the *potential* and *effective* production growth rates. The (s) rate might change erratically at $\{\mu: (0,1)\}$ influenced by *business cycles*. If at each instant the thrift level S_τ equals K_τ in the economy, by definition:

$$S_\tau = \mathrm{K}_\tau = F(sY_\tau) \quad (54)$$

and it "may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc." (Harrod, 1939, p. 16). Assuming that S_τ is used to finance K_τ of the value K_τ incorporated into Y_τ, another fraction of K_τ remains in the production process at each instant, say $\check{\mathrm{K}}_\tau$, then:

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$$(55)\ K_\tau = \check{K}_\tau + \kappa_\tau$$

And this must be equal to:

$$(56)\ K_\tau = \check{K}_\tau + S_\tau$$

The *effective capital accumulation* can be written as:

$$(57)\ \kappa_\tau = \underline{K}_\tau + \acute{K}_\tau$$

where $\underline{K}_\tau$ is the value of capital depreciated at τ, such that:

$$(58)\ K_\tau = \check{K}_\tau + \underline{K}_\tau + \acute{K}_\tau$$

It is reasonable that S_τ might differ from $\underline{K}_\tau$ due to new investments $\acute{K}_\tau$ at any given instant:

$$(59)\ S_\tau = \underline{K}_\tau + \acute{K}_\tau$$

Consequently:

$$(60)\ \acute{K}_\tau = S_\tau - \underline{K}_\tau$$

Of course: $1 = \frac{\check{K}_\tau}{K_\tau} + \frac{\underline{K}_\tau}{K_\tau}$ and let $\delta = \frac{\check{K}_\tau}{K_\tau}$ be the *remaining capital rate* at τ, and $(1-\delta) = \frac{\underline{K}_\tau}{K_\tau}$ is the *depreciation rate* and thus $[1 = \delta K_\tau + (1-\delta)K_\tau]$. To simplify $(\kappa_\tau - \acute{K}_\tau) = (1-\delta)K_\tau$ and $\check{K}_\tau = \delta K_\tau$. Introducing the interest rate (i) at which $S_\tau = F(sY_\tau)$ will be charged by the money market at each given instant, we get:

$$(61)\ S_\tau = (1+i)F(sY_\tau)$$

Defining $\frac{1}{1+i}S_\tau = F(sY_\tau)$ then:

$$(62)\ \kappa_\tau = F\left(\frac{1}{1+i}S_\tau\right)$$

This function explains that even depreciation could not be replaced at some instants: "(…) replacement falls far short of depreciation. Hence, investment net of depreciation cannot be identified with investment net of replacement" (Domar, 1953).

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By calling $\underline{k}$ the rate of capital accumulation with respect to the thrift level at each given instant, we redefine function (54) to state that K_τ could differ from $S = sY_\tau$ as follows:

$$(63)\ K_\tau = \underline{k}F(sY_\tau)$$

This leads to $\left(\underline{k} = \frac{1}{s}\frac{K_\tau}{Y_\tau} \therefore s\underline{k} = \frac{K_\tau}{Y_\tau}\right)$ and by equating these last two functions we get $\underline{k}sY_\tau = \frac{1}{1+i}S_\tau$ and thus:

$$(64)\ \underline{k} = \frac{1}{1+i}$$

which accounts for an inverse variation of the capital accumulation rate at i changes, but a direct variation at S changes. It seems clear by equations (53) and (64) that:

$$(65)\ K_\tau = F\left(\frac{1}{1+i}\frac{1-\alpha}{\alpha\mu + (1-\alpha)}\frac{\lambda}{\gamma}\right)Y_\tau$$

Differentiating function (62) in terms of S_τ such that $\frac{dS_\tau}{dY_\tau} = (1+i)s$ so that $\frac{\acute{s}}{\lambda}\frac{S_\tau}{Y_\tau} = (1+i)s$ and due to $s = \frac{S_\tau}{Y_\tau}$ we presume that:

$$(66)\ \acute{s} = (1+i)\lambda$$

$\acute{S}$ expresses the relative *potential* rate—or the *effective* rate after replacing λ with γ—of thrift as a function of the interest rate and production growth; *ceteris paribus*, at $i > 0;\ \acute{s} > \lambda\,\acute{k}$ will fall. Per equation (66) $\frac{\acute{s}}{\lambda} = (1+i)$, which inserted into equation (64) will give:

$$(67)\ \lambda = \acute{s}\underline{k}$$

This outcome explains that potential relative production growth will also depend on the relative rates of thrift and capital accumulation; we claim that $\acute{S}$ is a production growth accelerator.

An approach to national accounting

Net product can be deduced as $NY = Y_\tau - \underline{K}_\tau$, whence a fraction of thrift is available to finance $\acute{K}_\tau$. So, *net net product* can be expressed as

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$NNY = NY - \acute{K}_\tau = Y_\tau - (K_\tau + \acute{K}_\tau) = Y_\tau - S_\tau = C_\tau$ where (C_τ) equals public and private consumption. Due to $S_\tau = sY_\tau$ then $C_\tau = Y_\tau - sY_\tau$ ∴

$$(68)\ C_\tau = (1 - s)Y_\tau$$

Including taxes, $NNY = C_\tau = C_L + C_K$, and presuming s is an average rate:

$$(69)\ C_L = (1 - s)\ddot{Y}_L$$

$$(70)\ C_K = (1 - s)\ddot{Y}_K$$

And for the public and private thrift level:

$$(71)\ S_L = s\ddot{Y}_L$$

$$(72)\ S_K = s\ddot{Y}_K$$

and thus:

$$(73)\ C_\tau + S_\tau = Y_\tau = C_\tau + K_\tau$$

Additionally, profit rate can be estimated as:

$$(74)\ \acute{\rho} = [(1 - \beta) - (1 - \alpha)]$$

and profit level is:

$$(75)\ \rho = \acute{\rho} Y_\tau$$

and ρ is a fraction of $\ddot{Y}_K$, which we include in C_K.

This explains that both capital and labor are sources of profit, as a result of (κ) and (μ), which, as a law, models the global production processes of *contribution/distribution*, denoting technological and social *divergence/inequity*. In Cobb and Douglas' (1928) words: "the processes of distribution are modeled at all closely upon those of the production of value". In the real word, at *stagnant minimum wage*, there are reasons to suspect that β will be greater than the *real* share of labor in production growth, profit will rise and inequity will grow.

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CONCLUSIONS

In the introduction to this research, we affirmed that our aim was to provide a new *method of attack* for analyzing the contribution and share of resources to production growth in a dynamic economy and thus attempt to answer and resolve Cobb and Douglas' questions, suggestions and challenges, as well as those of Harrod. Our model is a set of formulas with which we measure *average* and *instantaneous* changes in production, capital and labor to compute the contribution to and share of capital and labor in production growth.

Production growth and the contribution to and share of resources depend on changes in production processes and market conditions. We provide formulas to calculate the elasticities of resource composition, resource contribution and resource share in production growth at each given instant. The slopes of the curves of contribution and share are also important results of this research. Making further assumptions allows us to find new formulas to estimate the rate and level of thrift required for the economy to continue production processes, and the rate and amounts of capital remaining at each instant, capital depreciated and new capital accumulated, and the rate of global capital accumulation.

A crucial conclusion is that $\left(\alpha = \beta = \frac{1}{2}\right)$ at $(\mu = \kappa = 1)$ and thus $\alpha\lambda = \alpha n$ and capital $(1-\alpha)\lambda = (1-\alpha)k$, stating the special case of *convergence*/*equity*; $(\lambda = n = k)$ such that $m = \kappa_\tau$. Even when $(\gamma \neq \lambda)$ due to technological changes $(\gamma_L = \gamma_K = \acute{y}_L = \acute{y}_K)$ and thus $(\dot{Y}_L = \dot{Y}_K = \ddot{Y}_L = \ddot{Y}_K)$, from which $\acute{\rho} = 0$. Our hypothesis is that no political system can obtain these results by simply planning economic developments, intending to regulate the law of production growth. The best a government can do is to provide reasonable macroeconomic and social policies to avoid divergence/inequity.

To evaluate the usefulness of our model, we utilized data from the United States of America (2012-2022)—which can be seen as a limitation of this study—the results and explanations of which are available in the Appendix. It falls on economic science to improve the method and replicate it in other scenarios in order to prove its strength in measuring and explaining the contribution and share of resources in the economic growth process. This research could be valuable for government and business decisions.

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Appendix

The United States economy (2012-2022)

This section provides an evaluation of the model proposed in this research by analyzing data from the United States of America (U.S.) for the years 2012-2022. By examining the dynamics of production, labor and capital according to the real value level index, we measured variables such as capital and labor productivity and share in production growth and thrift and capital accumulation and consumption, revealing the model's robustness. We justify application of the model to the U.S. economy given that it represents around 20 percent of total global output, and it is a highly developed and technologically advanced economy, with a large number of its corporations present at the international level (IMF, 2020).

Data and methodology

The figures for annual Gross Domestic Product (Y) and Gross Fixed Capital Formation (K)—structures, equipment and intangibles—come from the Bureau of Economic Analysis (BEA) expressed in U.S. dollars $(2012 = 100)$. Labor (L) represents full-time equivalent employees plus the number of self-employed persons; unpaid family workers are not included. The period of study begins in the year 2012 due to employee data only being available up to that year, which became the BEA's base year for measuring (Y, K) real values. Index numbers were built to apply the model and to provide evidence of its analytical and explanatory capacity.

Relevant results

The *effective* average *real* growth rate of U.S. product was 2.1 percent in 2013-2016[6], 2.3 percent in 2016-2019 and 1.9 percent in 2019-2022, which included COVID-19 and the Russian invasion of Ukraine. Respectively, employment growth was 1.54, 1.43 and 0.48 percent while the growth of capital was 3.4 percent in the first two periods and 1.73 percent in the last. The lower growth rates of labor relative to capital and the increase in production reveal an increment in labor productivity as resource composition κ rises whereas its elasticity μ falls. $\gamma > \lambda$, rising at $\mu{:}\left(\frac{1}{2}, 1\right)$ and falling at $\mu \geq 1$ (Table 1, Figure 2), could reflect the influence of technological changes on resource productivity, illustrated by the gap between $(Y, \hat{Y})$ (Table 1, Figure 1).

6 (γ) increments can be attributed to the Federal Reserve's (Fed) monetary stimulus during the 2007-2012 global economic crisis. In the 2008 financial crisis, the Fed deployed its unconventional *quantitative easing* (QE) program, purchasing longer-term securities from the open market. The theoretical objectives are that increasing money supply encourages cheaper lending and investment by forcing interest rates down with the ultimate goal of stimulating economic growth.

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Table 1. Relative Rates of Production, Capital and Labor Growth: Resource composition, elasticity of resource composition, contribution and distribution in the U.S. (2012-2022).

	Index level of:			Effective growth rate of:			Resource composition	Labor contribution based on κ	Elasticity of resource composition	Labor contribution based on μ	Potential production growth	Labor share	Potential production level
	Production	Labor	Capital	Production	Labor	Capital							
Year	Y	L	K	γ	n	k	κ	α	μ	α	λ	β	Ŷ
2012	100	100	100	-	-	-	1.0000	-	-	-	-	-	100
2013	102	101	104	0.0184	0.0102	0.0386	1.0281	0.4931	0.2653	0.7903	0.0162	0.7857	102
2014	104	103	112	0.0229	0.0165	0.0795	1.0918	0.4780	0.2076	0.8281	0.0273	0.8152	104
2015	107	104	115	0.0271	0.0173	0.0225	1.0974	0.4768	0.7694	0.5652	0.0195	0.5422	106
2016	109	106	114	0.0167	0.0175	-0.0047	1.0734	0.4823	3.6932	0.2131	0.0075	0.2014	107
2017	111	108	119	0.0224	0.0126	0.0465	1.1094	0.4741	0.2699	0.7875	0.0198	0.7696	109
2018	114	109	128	0.0295	0.0158	0.0688	1.1673	0.4614	0.2298	0.8132	0.0257	0.7885	112
2019	117	111	131	0.0229	0.0114	0.0264	1.1846	0.4577	0.4321	0.6983	0.0159	0.6614	114
2020	114	104	123	-0.0277	-0.0618	-0.0642	1.1817	0.4584	0.9634	0.5093	-0.0630	0.4676	107
2021	121	107	131	0.0595	0.0324	0.0691	1.2237	0.4497	0.4688	0.6808	0.0441	0.6355	111
2022	123	111	136	0.0208	0.0374	0.0379	1.2242	0.4496	0.9885	0.5029	0.0376	0.4525	116

Source: Based on data from the Bureau of Economic Analysis.

Daniel Villalobos Céspedes

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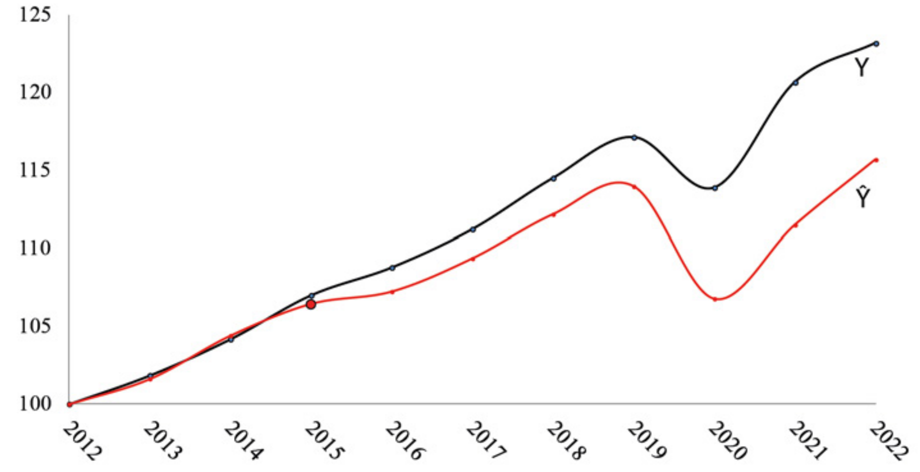
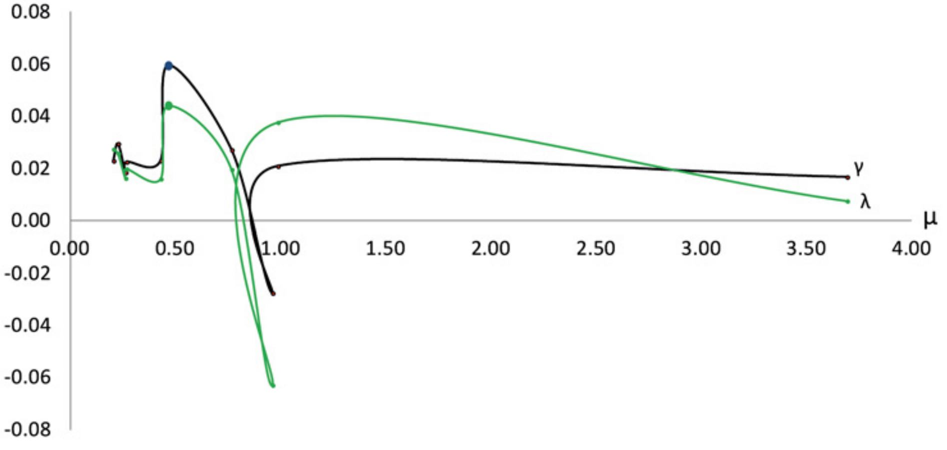
Figure 1. U.S. Economy: Effective and potential production trends (2012-2022).



Source: Based on Table 1.

Figure 2. U.S. Economy: Tendency of *effective* (γ) and *potential* (λ) production growth at μ (2012-2022).



Source: Based on Table 1.

The above results are coherent with the performance of (α, β), which both decline as $(\mu > 0;\ \mu\kappa > 0)$ (Table 1, Figure 3). $\alpha > \beta$ probably explains that labor was not compensated for their productivity, even though they could have taken a greater fraction of production growth as compared to capital (Table 2, Figure 4). So (α, β) are indicators of *convergence/divergence* from which *equity/inequity* results at each given instant, as a result of the structure of production processes, government (institutional)

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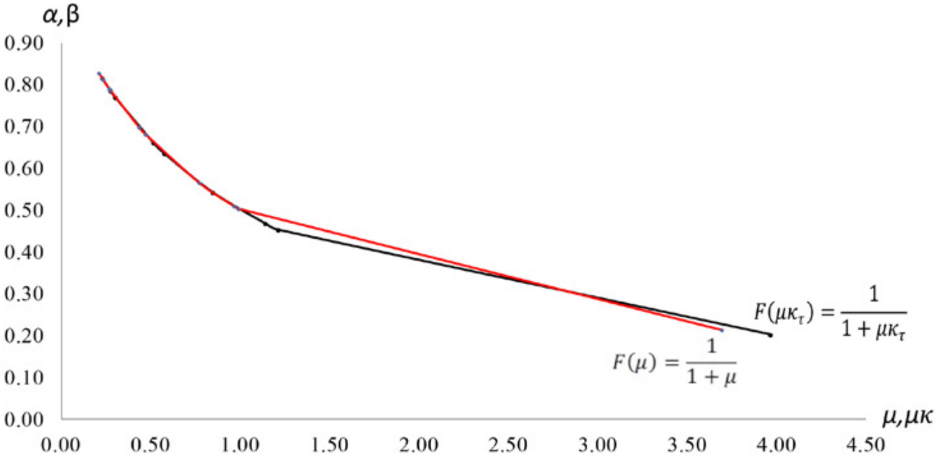
policies, market instability, labor shortage and wage growth, productivity and natural and international events. It seems that *divergence*/*inequity* rise during times of crisis, as could be the case in the U.S. economy in the years 2016, 2020 and 2022; both capital and labor rise insufficiently (Table 1) and labor contribution and share fall much more than capital when $\mu \geq 1$ or becomes highly volatile (Table 2, Figure 5).[7]

Figure 3. U.S. Economy: Tendency of the elasticity of resource contribution to (α) and share (β) in production growth at $(\mu, \mu\kappa)$ (2012-2022).



Source. Based on Table 1.

7 This is consistent with the Fed's report (2023).

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Table 2. U.S. Economy: Effective capital and labor contribution to and share in production growth (2012-2022).

	Effective contribution			Production level	Total contribution		Effective share			Production level	Total share	
	With α based on μ Villalobos (2019, 2019a, 2020, 2020a and this research)						Based on β in this research					
Year	y_L	y_K	y	$\boxtimes$	$\boxtimes_K$	$\boxtimes_L$	$\acute{y}_L$	$\acute{y}_K$	$\dot{y}$	$\boxtimes$	$\ddot{Y}_L$	$\ddot{Y}_K$
2012	-	-	-	100	-	-	-	-	-	100	-	-
2013	1.46	0.39	1.84	102	21	80	1.45	0.39	1.84	102	80	22
2014	1.93	0.40	2.33	104	18	86	1.90	0.43	2.33	104	85	19
2015	1.59	1.23	2.82	107	47	60	1.53	1.29	2.82	107	58	49
2016	0.38	1.40	1.78	109	86	23	0.36	1.42	1.78	109	22	87
2017	1.92	0.52	2.44	111	24	88	1.88	0.56	2.44	111	86	26
2018	2.66	0.61	3.28	114	21	93	2.58	0.69	3.28	114	90	24
2019	1.83	0.79	2.63	117	35	82	1.74	0.89	2.63	117	77	40
2020	-1.65	-1.59	-3.24	114	56	58	-1.52	-1.73	-3.24	114	53	61
2021	4.61	2.16	6.77	121	39	82	4.30	2.47	6.77	121	77	44
2022	1.26	1.25	2.51	123	61	62	1.14	1.38	2.51	123	56	67

Source: Based on Table 1.

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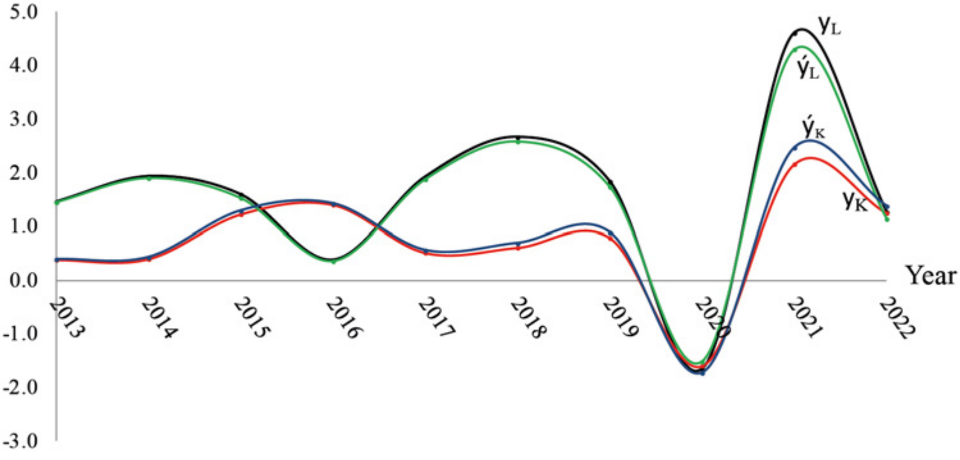
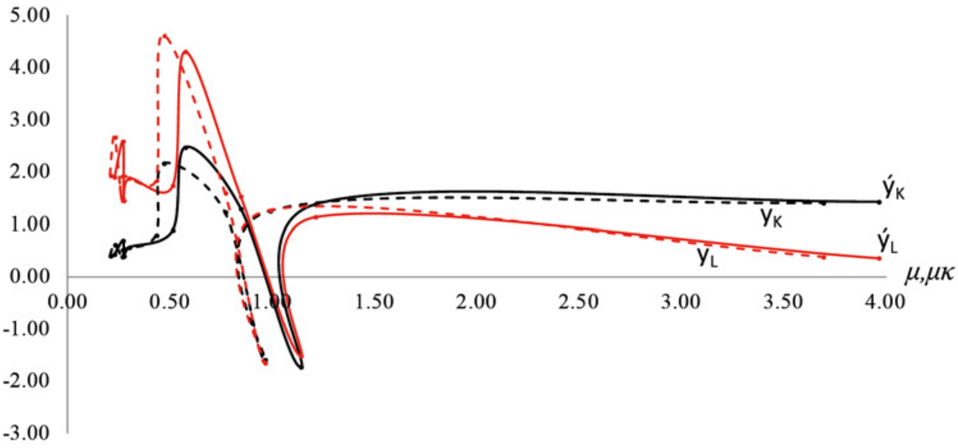
Figure 4. U.S. Economy: Effective contribution to and share of labor and capital in production growth (2012-2022).



Source: Based on Table 2.

Figure 5. U.S. Economy: Labor and capital contribution to (α) and share (β) in production growth at $(\mu, \mu\kappa)$ (2012-2022).



Source: Based on Table 2.

Except for the years 2016, 2020 and 2022, the rate of thrift (s) increases at a somewhat stable level at $\left[\mu: \left(\frac{1}{2}, 1\right)\right]$ and decreases at $(\mu > 1)$. Moreover (s, γ) rose at $\left[\beta: \left(\frac{1}{2}, 1\right)\right]$ precisely when $\left[\mu\kappa: \left(\frac{1}{2}, 1\right)\right]$. This also coincides with a stable thrift level

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(S) in the years in which it was supposed that labor income $\ddot{Y}_L > 70$ percent of the product except in 2020 and 2022, while (s, ρ) rose; the greater (α, β) the higher (s, ρ) as a tendency and $(\underline{K}, \acute{K}, \kappa)$ are at their highest values in the period, perhaps as an effect of the low interest rates[8] induced by the Fed's stimulus starting in 2012 (Table 3).

In 2015-2016, the Fed tightened its expansionary monetary policy inducing interest rates to rise[9], which could stagnate depreciation $\underline{K}$ and reduce new investments $\acute{K}$.[10] In these years $\ddot{Y}_L$ reached its lowest, most distant point from $\ddot{Y}_K$ in 2016 and thrift remained low, tending downward for the rest of the period, whereas ρ rose.[11] In 2017-2019, as the U.S. economy recovered its path of growth with $\ddot{Y}_L \cong 80$, however, perhaps the low interest rate kept the rate of thrift in decline while $\acute{K}$ rose, but $\underline{K}$ remained below prior-2012-2014 levels, reflected by $\kappa \cong 50$ stagnant at lower levels even though ρ grew strong and the stock of idle capital $\check{K}$ was higher than previous years (Tables 3 and 4).[12]

8 Since mid-2010, interest rates continued to fall approaching zero and started rising in late-2015 reaching 3.0 percent in 2019, at which point they began to fall again (IMF, 2020).

9 The Fed (2016) reports that labor market conditions continued to improve during the second half of 2015 and into early 2016; inflation was low (2%) and real GDP was increasing, but domestic financial conditions have become somewhat less supportive of economic growth since mid-2015.

10 The Fed (2017) reports that real outlays for business investment—private nonresidential fixed investment—were generally weak in 2016 and widespread across categories of equipment investment. Investment in equipment and intangibles tended downward over most of the year.

11 The Fed reported (2018) that with spending growth estimated to have outpaced income growth, the personal thrift rate has declined considerably since the end of 2015.

12 These results were probably induced by uncertainty regarding trade tensions and the weak global growth outlook (Fed, 2020) and in spite of expansionary fiscal policy and QE.

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Table 3. U.S. Economy. Thrift and capital accumulation (2012-2022).

	Thrift rate	Thrift level	Remaining capital	Remaining capital rate	Capital depreciation	New capital accumulation	Effective capital accumulation	Capital stock
Year	s	S	⊠	δ	⊠	⊠	⊠	K
2012	-	-	-	-	-	-	-	100
2013	0.4397	45	59	0.5908	41	4	45	104
2014	0.5975	62	50	0.4802	54	8	62	112
2015	0.3610	39	76	0.6780	36	3	39	115
2016	0.2234	24	90	0.7833	25	-1	24	114
2017	0.4409	49	70	0.6167	44	5	49	119
2018	0.4364	50	78	0.6503	42	8	50	128
2019	0.3472	41	90	0.7078	37	3	41	131
2020	1.1380	130	-7	-0.0535	138	-8	130	123
2021	0.3707	45	86	0.7042	36	8	45	131
2022	0.9040	111	25	0.1883	106	5	111	136

Source: Based on Table 1.

From our approach to national accounting, without detailing taxes (and for this government receipts and expenditures) and the foreign sector, an essential outcome is that in 2015 and 2016, labor thrift S_L dropped to the lowest level of the period while capital thrift S_K reached its highest value and the rate of profit $\acute{\rho}$ was higher than in previous years (Table 4). Moreover, C_L fell while C_K rose and as a whole $C_{U.S}$ struck its highest levels. After 2016, except in the years 2020 and 2022, consumption increased at levels not before seen while thrift slowed down.

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Table 4. U.S. Economy. Capital and labor consumption and thrift (2012-2022).

	Net U.S. Product	Net *net* U.S. product equals consumption	Consumption of:		Thrift level of:		Profit:	
			Labor and government	Capital and government	Labor and government	Capital and government	Rate	Level
Year	NY	$NNY = C$	C_L	C_K	S_L	S_K	$\dot{\rho}$	ρ
2012	-	-	-	-	-	-	-	-
2013	61	57	45	12	35	10	0.0046	0.4713
2014	50	42	34	8	51	12	0.0129	1.3406
2015	71	68	37	31	21	18	0.0230	2.4568
2016	84	84	17	67	5	19	0.0116	1.2662
2017	67	62	48	14	38	11	0.0179	1.9906
2018	73	65	51	14	39	11	0.0246	2.8222
2019	80	76	51	26	27	14	0.0368	4.3152
2020	-24	-16	-7	-8	61	69	0.0417	4.7466
2021	84	76	48	28	28	16	0.0454	5.4728
2022	17	12	5	6	50	61	0.0504	6.2098

Source: Based on Table 1.

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